

# Protons and Path Integrals

## Landmark Simulation of Condensed Phase Proton Transfer

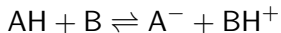
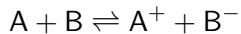
Thomas Allen (PI: Nancy Makri)

Department of Chemistry  
University of Illinois

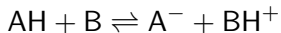
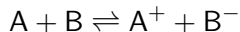
May 13, 2015

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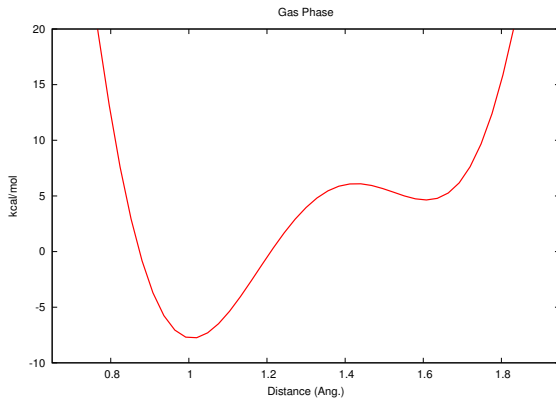
- These reactions are ubiquitous in biology
- Transfer of H, H<sup>+</sup>, and H<sup>-</sup> is a major synthetic motif
- Cutting-edge materials for energy storage and transport

# The Proton Transfer Problem

- Proton transfer is a condensed phase process

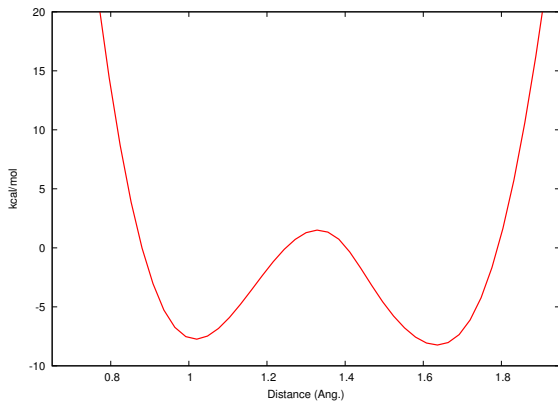
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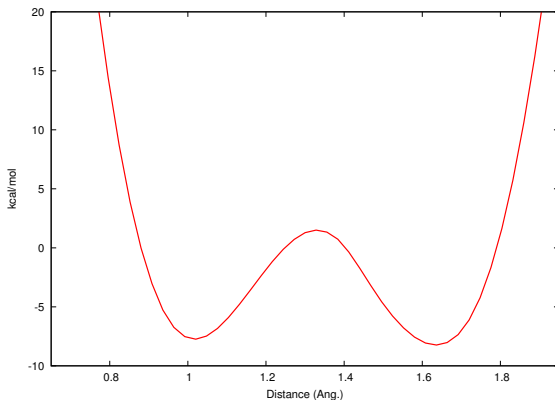
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- Many degrees of freedom, transferring species is quantum mechanical
- Separation into interacting system and environment is key



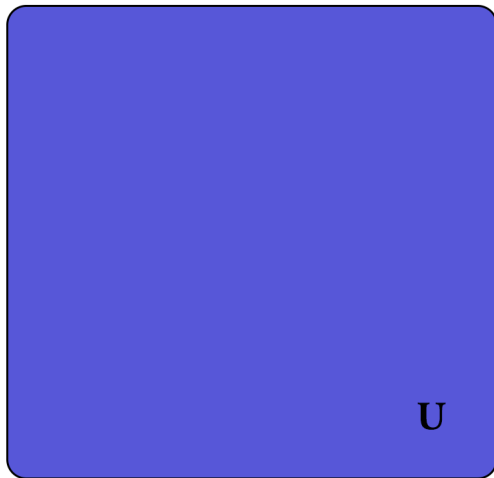
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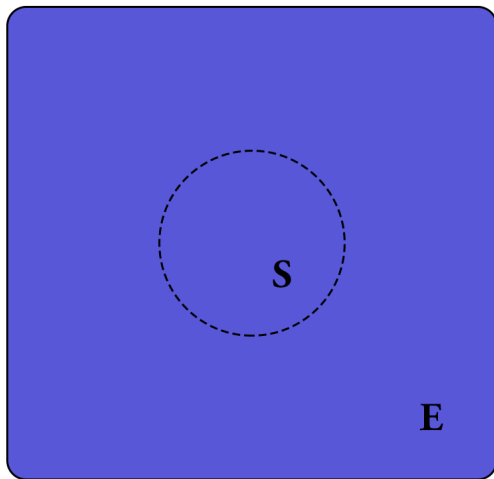
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  - Reduced Models (Spin-Boson, etc.)

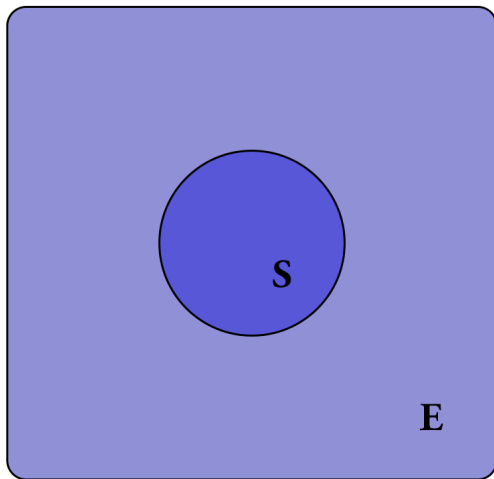
# Quantum-Classical Approaches

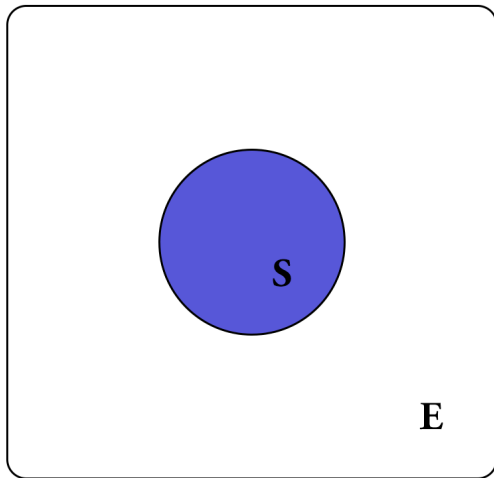
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- We desire a rigorous method that works across many regimes of behavior
- Capturing full system-bath interaction is especially important
- The Quantum-Classical Path Integral formalism is designed to achieve these goals











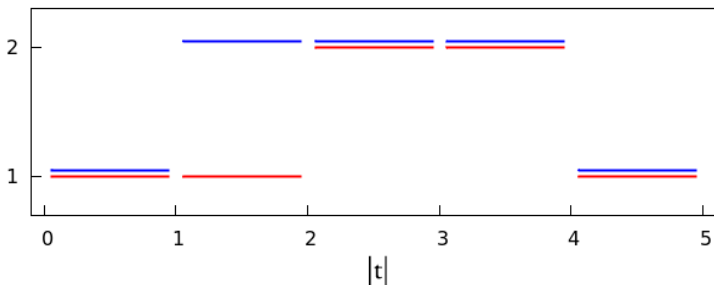
$$\hat{\rho}_{red}(s_N^\pm; N\Delta t) = \int dx_N^\pm \langle s_N^+ x_N^+ | e^{-i\hat{H}N\Delta t/\hbar} \hat{\rho}(0) e^{i\hat{H}N\Delta t/\hbar} | s_N^- x_N^- \rangle$$

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R. Lambert, N. Makri, *J. Chem. Phys.* **137**, 22A552 and 22A553 (2012)

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H. Azzouz, D. Borgis, *J. Chem. Phys.* **98**, 7361 (1993)

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- A test system for our method should have several properties
  - Simple MD description
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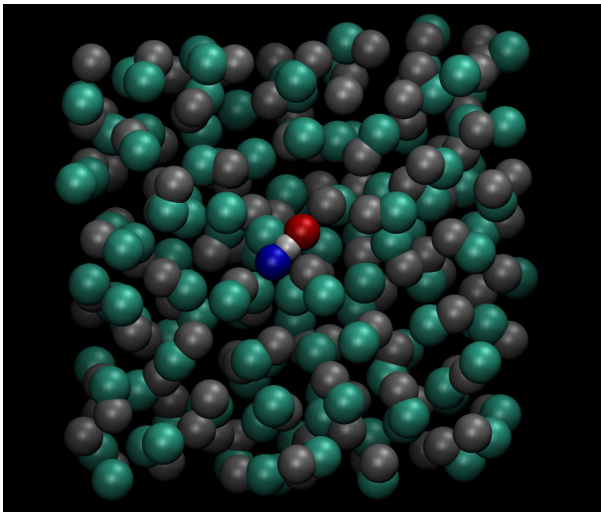
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- Our goal is to extend previous work to treat atomistic systems
- A test system for our method should have several properties
  - Simple MD description
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  - Rigorous approach is beneficial
- The Azzouz-Borgis model of proton transfer is just such a system

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# The Azzouz-Borgis Model



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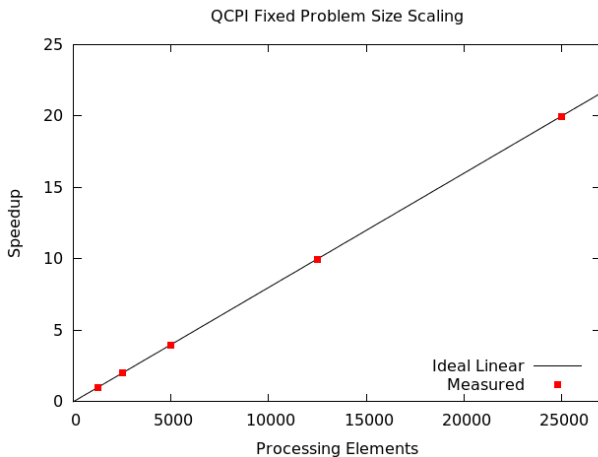
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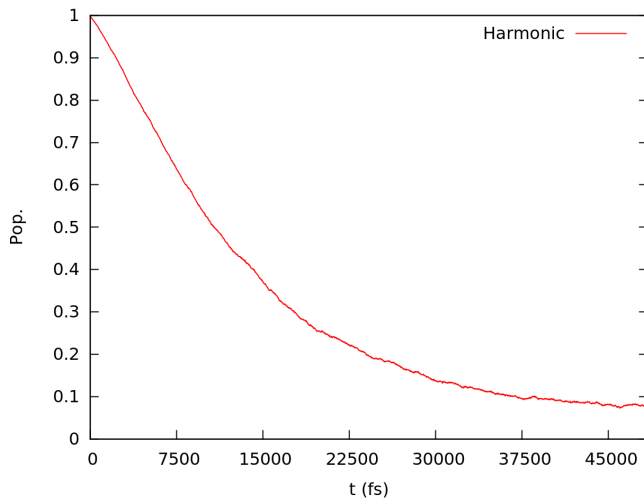
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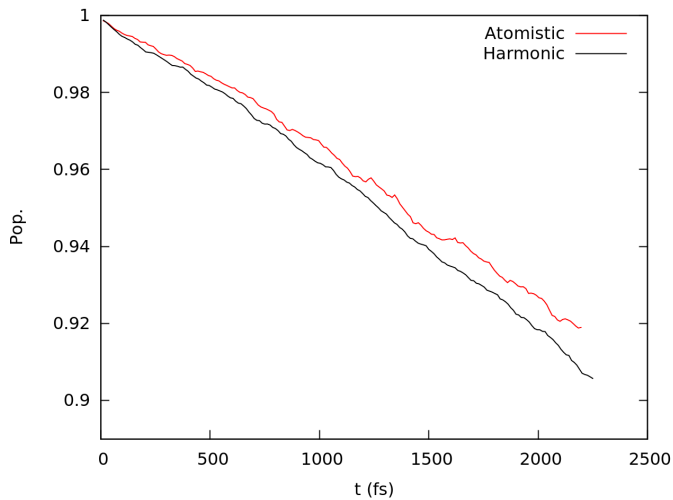
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- Further refinements suggested by BW staff
  - Using memory for file storage
  - Investigating multi-level parallelism







- Complete converged anharmonic calculations
- Investigate bath ensemble properties
- Extending results to complex systems, including proteins and biomolecules
  - Although these systems are larger, their couplings may be more manageable



# Acknowledgements

